

Conservation of Energy

- 1) A cannon ball is fired from Earth's surface. At the peak of its trajectory, it has a horizontal speed of 160 m/s and is 1.20×10^3 m above the ground. With reference to the ground, what is the mechanical energy of the cannon ball at the highest point on its trajectory, if the mass of the cannon ball is 5.20 kg? [1.28×10^5 J]

$$E_m = mgh + \frac{1}{2} mv^2$$

- 2) A rocket is accelerating upward. When the rocket has reached an altitude of 5.00×10^3 m, it has reached a speed of 5.40×10^3 km/h. Relative to its launch site, what is its mechanical energy, if the mass of the rocket is 6.50×10^4 kg? [7.63×10^{10} J]

$$E_m = mgh + \frac{1}{2} mv^2 \quad // \text{Remember to convert km/h into m/s!}$$

- 3) What is the speed of a 4.50-kg cannon ball if, at a height of 275 m above the ground, its mechanical energy relative to the ground is 6.27×10^4 J? [150 m/s]

$$E_m = mgh + \frac{1}{2} mv^2 \quad // \text{Solve for } v$$

- 4) As a roller coaster trolley with a mass of 600 kg coasts down the first hill, it drops a vertical distance of 45.0 m from an initial height of 51.0 m above the ground. If, at the bottom of the hill, its speed is 30.0 m/s:

- i) what is the trolley's mechanical energy relative to the top of the hill, and [5.13×10^3 J]

$$E_m = mgh + \frac{1}{2} mv^2 \quad // \text{Use height from top of hill}$$

- ii) what is the trolley's mechanical energy relative to the ground? [3.05×10^5 J]

$$E_m = mgh + \frac{1}{2} mv^2 \quad // \text{Use height above ground}$$

- 5) A mass of 55.0 kg is 225 m above the ground with a velocity of 36.0 m/s [down]. Use conservation of energy to calculate its velocity when it reaches a height of 115 m above the ground. Ignore the effects of air resistance. [58.8 m/s [down]]

$$mgh + \frac{1}{2} mv^2 = mgh' + \frac{1}{2} mv'^2 \quad // \text{Solve for } v'$$

- 6) A 70 kg diver jumps from a 12 m tower, with no initial velocity.

- i) With what speed does the diver hit the water? [15 m/s]

$$mgh = \frac{1}{2} mv^2 \quad // \text{Solve for } v$$

- ii) What would his impact velocity be if, in jumping from the tower, he gave himself an upward initial velocity of 5.0 m/s? [16 m/s]

$$mgh + \frac{1}{2} mv^2 = mv'^2 \quad // \text{Solve for } v'$$

- 7) What upward velocity must an Olympic high jumper impart to herself in order to clear the 2.0 m height of the bar? Assume she must raise her centre of mass to just above the bar, and that her centre of mass is 0.85 m above ground level when she begins to jump. [4.7 m/s]

$$mgh + \frac{1}{2} mv^2 = mgh' \quad // \text{Solve for } v$$

- 8) A human "cannon ball" in the circus is shot at a speed of 21.0 m/s at an angle of 20° above the horizontal from a platform that is 15.0 m above the ground.

- i) If the acrobat has a mass of 56.0 kg, what is his gravitational potential energy relative to the ground when he is at the highest point of his flight? Ignore the effects of air resistance. [9.69×10^3 J]

$$mgh + \frac{1}{2} mv^2 = E_p' + \frac{1}{2} mv_x^2 \quad // \text{Solve for } E_p'$$

- ii) If the net in which he lands is 2.00 m above the ground, how fast is he travelling when he hits it? [26.4 m/s]

$$mgh + \frac{1}{2} mv^2 = mgh' + \frac{1}{2} mv'^2 \quad // \text{Solve for } v'$$

9) A model rocket has a mass of 3.00 kg. It is fired so that when it is 220 m above the ground it is travelling vertically upward at 165 m/s. At that point its fuel runs out so that the rest of its flight is without power. Assume that the effect of air friction is negligible and that all potential energies are measured from the ground.

- i) What is the mechanical energy of the rocket, relative to the ground, when it is 220 m above the ground? [4.73×10^4 J]

$$E_m = mgh + \frac{1}{2} mv^2$$

- ii) When it reaches the highest point on its trajectory, what will its gravitational potential energy be? [4.73×10^4 J]

$$mgh + \frac{1}{2} mv^2 = E_p' \quad // \text{ Solve for } E_p'$$

- iii) How far above the ground is the rocket at its highest point? [1.61×10^3 m]

$$mgh + \frac{1}{2} mv^2 = mgh' \quad // \text{ Solve for } h'$$

- iv) When it hits the ground, what is its speed? [178 m/s]

$$mgh + \frac{1}{2} mv^2 = \frac{1}{2} mv'^2 \quad // \text{ Solve for } v'$$

10) A pole-vaulter with a mass of 56.0 kg tries to convert the kinetic energy of her approach into height.

- i) What is the maximum height she can expect to attain if her approach speed is 8.00 m/s? Assume that the centre of mass of the vaulter is initially 0.850 m above the ground. [4.11 m]

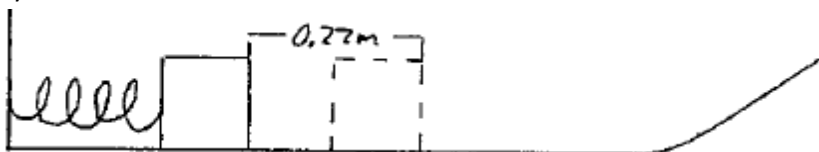
$$\frac{1}{2} mv^2 + mgh = mgh' \quad // \text{ Solve for } h'$$

- ii) Describe the energy changes that occur from the time the vaulter starts to run until she reaches the highest point of her jump.

11) A ball bearing of mass 50 g is sitting on a vertical spring whose force constant is 120 N/m. By how much must the spring be compressed so that, when released, the ball rises to a maximum height of 3.1 m above its release position? [0.16 m]

$$\frac{1}{2} kx^2 = mgh \quad // \text{ Solve for } h \text{ and remember to convert mass into kg}$$

12) A 2.0 kg mass is placed against a spring of force constant 800 N/m, which has been compressed 0.22 m, as illustrated. The spring is released, and the object moves along the horizontal frictionless surface and beyond. Calculate:



- i) The maximum elastic potential energy of the spring. [19 J]

$$E_p = \frac{1}{2} kx^2 \quad // x = 0.22\text{m}$$

- ii) The maximum velocity of the mass. [4.4 m/s]

$$\frac{1}{2} kx^2 = \frac{1}{2} mv'^2 \quad // \text{ solve for } v'$$

- iii) The maximum vertical height of the mass up the slope. [1.0 m]

$$\frac{1}{2} kx^2 = mgh \quad // \text{ solve for } h$$

13) A bullet's speed may be determined by firing it into a sandbag pendulum, and measuring the vertical height to which the pendulum rises. The bullet stays in the sandbag. A 20.0 g bullet strikes a 10.0 kg sandbag and causes it to rise vertically 0.040m.

- i) What is the change in gravitational potential energy of the sandbag and bullet during the swing? [3.9 J]

$$E_p = (m_b + m_s)gh$$

ii) What is the velocity of the sandbag-bullet combination at the start of the swing? [0.89 m/s]

$$\frac{1}{2} (m_b + m_s) v_{bs}^2 = (m_b + m_s) gh \quad // \text{ Solve for } v_{bs}$$

iii) What is the original velocity of the bullet? [4.5×10^2 m/s]

// Have to use conservation of momentum as inelastic collision means energy is not conserved

$$p = p'$$

$$m_b v_b = (m_b + m_s) v_{bs} \quad // \text{ solve for } v_b$$